



An interval type-2 fuzzy sets-based TODIM method and its application to green supplier selection

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With the increasing awareness and significant environmental pressures from various stakeholders, companies have realized the significance of selecting green suppliers to their supply chain activities, which involves multiple criteria with uncertainty and the decision makers' behaviour with irrational. Interval type-2 fuzzy sets (IT2 FSs) have advantages in modelling uncertainty over type-1 fuzzy sets. And TODIM is an useful non-linear prospect model for selecting the irrationally determined alternatives, but the ratings and weights are crisp values. In this paper, we develop the IT2 FSs-based TODIM method to select green supplier. First, we introduce a new distance computing method for IT2 FSs to assist the dominance models to deal with gains (losses) computation. Second, we identify the gains (losses) computing expression through comparing the ranking values of the IT2 FSs evaluations, and obtain the dominance degree of one alternative over others. Third, we use the presented IT2 FSs ranking method using possibility mean and variation coefficient concepts to defuzzify the dominance degree, and obtain the crisp global performance to select the best alternative. Finally, we also apply the proposed IT2 FSs-based TODIM method to green supplier selection for automobile manufacturers.

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1. Introduction

With the governmental legislation and an increased awareness for protecting the environment, companies should not overlook environmental issues if they want to keep their long-term advantage in contemporary competitive environment. They must offer their products and services with proper quality, price, speed and environment protection to the customers (Seuring, 2013). Consequently, it is necessary to consider environmental pollution issues that appearers in supply chain activities, namely, green supply chain management (GSCM) (Hsu and Hu, 2008; Diabat and Govindan, 2011; Hsu *et al.*, 2013). In recent years, companies have implemented several environmental protection, resource conservation and the sustainable development policy, which ensure that suppliers can provide materials and services both satisfying stringent environmental quality and performance criteria (Awasthi *et al.*, 2010). Green suppliers selection should be considered primary and important when firms look for GSCM (Feng, 2012). Up to now, there are at least five comprehensive reviews on the advances of green supplier selection problems (Srivastava, 2007; Sarkis *et al.*, 2011; Genovese *et al.*, 2013; Igarashi *et al.*, 2013; Fahimnia *et al.*, 2015). Numerous multiple criteria decision-making (MCDM) methods have been analysed for green supplier evaluation and

selection problems. Kuo *et al.* (2010) integrated artificial neural network and MCDM methods for green supplier selection. James and Tao (Freeman and Chen, 2015) used the an AHP–Entropy–TOPSIS framework to select green suppliers. Kuo *et al.* (2015) proposed a novel hybrid MCDM model to evaluate green suppliers in an electronics company. Hu *et al.* (2015) presented an optimization decision method for green supplier selection under the mode of low carbon economy.

However, for green supplier selection problems some criteria are usually not known precisely, especially for the environment factors, such as green design capability and use of green materials in the production process. In these cases, the theory of fuzzy sets is one of the best tools for handling uncertainty, and the values of criteria are usually represented by type-1 fuzzy sets (T1 FSs) (Saen, 2009; Liu *et al.*, 2012; Sen *et al.*, 2014). Several type-1 fuzzy MCDM methods have been proposed for dealing with green supplier selection problems. Lee *et al.* (2009) proposed type-1 fuzzy MCDM green supplier selection model for high-tech industry. Kannan *et al.* (2013) introduced an integrated type-1 fuzzy AHP and TOPSIS approach for green supplier selection problem. Kannan *et al.* (2015) proposed a fuzzy axiomatic design model to select the best green supplier for Singapore-based plastic manufacturing company.

The membership value of T1 FSs is a crisp number in $[0, 1]$. However, it is usually hard to find out the exact membership function for a fuzzy set. Later, Zadeh (1975) proposed type-2 fuzzy sets (T2 FSs) (Mendel, 2001), which are described by

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both primary and secondary membership to provide more degrees of freedom and flexibility, and their membership functions are represented by a fuzzy set on the interval $[0, 1]$. Hence, T2 FSs are more accurate in the modelling of uncertainty in comparison with T1 FSs. Nevertheless, because of the variability of the primary membership value, T2 FSs faced heavy computations when used to solve MCDM problems (Liu, 2008). Then, interval type-2 fuzzy sets (IT2 FSs) are introduced with some representations such as vertical-slice and wavy-slice representations (Zadeh, 1975; Liu, 2008). It is defined by an interval-valued membership function, which contain membership values that are crisp intervals and extremely advantageous for theoretical and computational studies of the higher-order fuzzy sets because of their relative simplicity (Mendel, 2001). Namely, if the meanings of the criteria are not clear, and the evaluations do not have the same pinions, IT2 FSs are able to offer effective decision support (Celik *et al.*, 2015). IT2 FSs have been found useful to handle vagueness and uncertainty in decision problems, such as location choice (Han and Mendel, 2012; Cebi and Otay, 2014; Wang and Chen, 2014), project test and selection (Wu and Mendel, 2010; Kilic and Kaya, 2015), investment decision (Cebi and Kahraman, 2010; Mendel and Wu, 2010; Celik *et al.*, 2013), transportation systems evaluation and optimization (Kundu *et al.*, 2014; Liu *et al.*, 2014; Jindong Qin, 2015; Kundu *et al.*, 2015; Sun *et al.*, 2015), operation management and e-commerce performance evaluation (Singh and Benyoucef, 2011; Gong, 2013; Sang *et al.*, 2015; Wang *et al.*, 2015).

On the other hand, decision makers' psychological behaviour is seldom considered in existing green suppliers selection models. TODIM (Gomes and Lima, 1992) is one of the first MCDM methods based on prospect theory, which uses prospect function to calculate the dominance of one alternative over another, and the ratings and weights are given as crisp numbers. Under this condition, Gomes and Rangel (2009) and Gomes *et al.* (2009) applied the TODIM method to the rental evaluation of residential properties and the best destination of the natural gas reserves. Gomes and González (2012) and Gomes *et al.* (2013) also generalized the TODIM method towards cumulative prospect theory and rewrote the dominance of TODIM method using Choquet integral. Fan *et al.* (2013) developed an extended TODIM method for solving the hybrid MCDM problem. Pereira *et al.* (2013) analysed the robustness of TODIM-based multi-criteria evaluation model. Lourenzutti and Krohling (2014) adapted the TODIM method through Hellinger distance and stochastic dominance degree.

The fuzzy set theory introduced by Zadeh (1965) and the concept of fuzzy numbers introduced by Dubois and Prade (1978) can be applied to improve the evaluation and weights of these factors for human judgments in MCDM problems (Hu *et al.*, 2014). Krohling *et al.* (2013) and Lourenzutti and Krohling (2013) generalized the Fuzzy-TODIM method to deal with intuitionistic fuzzy information. Zhang and Xu (2014) extended the TODIM method to solve MADM problems under

hesitant fuzzy environment. Passos *et al.* (2014) introduced the TODIM-fuzzy synthetic evaluation method to solve classification problems. Tosun and Akyüz (2015) proposed fuzzy TODIM method and applied it to the supplier selection problem. Tseng *et al.* (2014) used fuzzy TODIM method to evaluate green supply chain practices. It is obvious that TODIM is one of the most valuable methods to solve MCDM problems considering decision makers' behaviour, and its extensions can also effectively solve the MCDM problems under fuzzy preference information. However, none studies have used TODIM method to handle the multiple criteria green supplier selection problems under IT2 FSs environment.

Although a considerable number of MCDM methods have already been proposed, there is still a need to develop the MCDM approach for handling ambiguity evaluations and irrationality of the decision makers in green supplier selection problems. In view of the characteristics of IT2 FSs and TODIM method, this paper proposes an IT2 FSs-based TODIM method to handle the multiple criteria green supplier selection problems. We first introduce the distance computation method for IT2 FSs, with which the distance between evaluations in gains (losses) expressions can be determined. Then, we choose the gains (losses) expression for the benefit and cost criteria through comparing the ranking values of IT2 FSs evaluations, and obtain the dominance values with IT2 FSs form. Once more, we use the IT2 FSs ranking method involving possibility mean and variation coefficient concepts to defuzzify the dominance degree and finally obtain the crisp performance for the alternatives. Finally, the proposed IT2 FSs-based TODIM approach is applied to solve green supplier selection decision problems, and the analysis with different parameter values of which is discussed. In conclusion, it provides new ways for developing TODIM method both in theory and its application.

The paper is organized as follows. Section 2 introduces the concept of IT2 FSs, the ranking method for IT2 FSs and the development of TODIM method. Section 3 proposes the distance computing method for IT2 FSs, and the fuzzy TODIM method under IT2 FSs environment. Section 4 illustrates an application in automobile green supplier evaluation and selection problem, and the sensitivity analysis is discussed with different parameter values. Section 5 summarizes the main results and draws conclusions.

2. Preliminaries

In this section, we introduce the concepts of IT2 FSs, the ranking method for IT2 FSs and the development of TODIM method.

2.1. Interval type-2 fuzzy sets

The T2 FSs are characterized by a fuzzy membership function, where the membership values are fuzzy sets in $[0, 1]$, not crisp numbers.

Definition 1 (Zadeh, 1975) The T2 FSs are represented by a type-2 membership function $\mu_{\tilde{A}}$, which can be shown as:

$$\tilde{A} = \frac{\int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u)}{(x, u)} = \frac{\int_{x \in X} \left[\int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \right]}{x},$$

where x is the primary variable, $J_x \in [0, 1]$ is the primary membership of x , u is the secondary variable and $\int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u)$ is the secondary membership function at x .

Mendel (2001) further generalized the interval fuzzy set and defined the notion of IT2 FSs, which is defined as follows.

Definition 2 (Mendel, 2001) The IT2 FSs \tilde{A} is an objective, which has the parametric form as:

$$\tilde{A} = \frac{\int_{x \in X} \int_{u \in J_x} 1}{(x, u)} = \frac{\int_{x \in X} \left[\int_{u \in J_x} 1 / (x, u) \right]}{x},$$

where x is the primary variable, $J_x \in [0, 1]$ is the primary membership of x , u is the secondary variable and $\int_{u \in J_x} 1 / (x, u)$ is the secondary membership function at x .

Take the interval type-2 trapezoid fuzzy number $\tilde{A} = [(a^U, b^U, c^U, d^U; h^U), (a^L, b^L, c^L, d^L; h^L)]$ as an example, the membership functions of which are defined as below:

$$\mu_{\tilde{A}^U}(x) = \begin{cases} (x - a^U) / (b^U - a^U), & x \in [a^U, b^U], \\ h^U, & x \in [b^U, c^U], \\ (d^U - x) / (d^U - c^U), & x \in [c^U, d^U], \\ 0, & \text{otherwise.} \end{cases}$$

$$\mu_{\tilde{A}^L}(x) = \begin{cases} (x - a^L) / (b^L - a^L), & x \in [a^L, b^L], \\ h^L, & x \in [b^L, c^L], \\ (d^L - x) / (d^L - c^L), & x \in [c^L, d^L], \\ 0, & \text{otherwise.} \end{cases}$$

where the upper membership value $h^U \in (0, 1]$, the lower membership value $h^L \in (0, 1]$ and $h^U \geq h^L$.

According to Zadeh's (1965) extension principle, the fuzzy set \tilde{A} can be denoted by its intervals as below.

Definition 3 (Mendel, 2001) For the IT2 FSs \tilde{A} , the footprint of uncertainty of \tilde{A} ($FOU(\tilde{A})$) is defined as:

$$\begin{aligned} FOU(\tilde{A}) &= \bigcup_{x \in X} J_x = \{(x, y) : y \in J_x = [\tilde{A}^U(x), \tilde{A}^L(x)]\}, \\ &= \bigcup_{x \in X} J_x = \{(x, y) : y \in J_x \\ &= [(\underline{A}^U(x), \bar{A}^U(x)), (\underline{A}^L(x), \bar{A}^L(x))]\}, \end{aligned}$$

where FOU is indicated as the shaded region. It is bounded by an upper membership function (UMF) $\tilde{A}^U(x)$ and a lower membership function (LMF) $\tilde{A}^L(x)$, both of which are T1 FSs. $\underline{A}^U(x)$ ($\underline{A}^L(x)$) and $\bar{A}^U(x)$ ($\bar{A}^L(x)$) are the left and right limit membership functions of $\tilde{A}^U(x)$ ($\tilde{A}^L(x)$). An example of IT2 FSs is shown in Figure 1.

Definition 4 Let \tilde{A} and \tilde{B} be two non-negative IT2 FSs, where $\tilde{A} = [(a^U, b^U, c^U, d^U; h^U), (a^L, b^L, c^L, d^L; h^L)]$ and $\tilde{B} = [(e^U, f^U, g^U, k^U; h^U), (e^L, f^L, g^L, k^L; h^L)]$.

The arithmetic operations between A and B are defined as follows.

(1) Addition operation

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= [(a^U + e^U, b^U + f^U, c^U + g^U, d^U + k^U; h^U), \\ & (a^L + e^L, b^L + f^L, c^L + g^L, d^L + k^L; h^L)] \end{aligned} \quad (1)$$

(2) Multiplication by a crisp number

$$\begin{aligned} \lambda \cdot \tilde{A} &= \tilde{A} \cdot \lambda \\ &= \begin{cases} [(\lambda a^U, \lambda b^U, \lambda c^U, \lambda d^U; h^U), (\lambda a^L, \lambda b^L, \lambda c^L, \lambda d^L; h^L)], & \text{if } \lambda \geq 0; \\ [(\lambda d^U, \lambda c^U, \lambda b^U, \lambda a^U; h^U), (\lambda d^L, \lambda c^L, \lambda b^L, \lambda a^L; h^L)], & \text{if } \lambda < 0. \end{cases} \end{aligned} \quad (2)$$

(3) Division by a non-zero number

$$\frac{\tilde{A}}{\lambda} = \begin{cases} \left[\left(\frac{a^U}{\lambda}, \frac{b^U}{\lambda}, \frac{c^U}{\lambda}, \frac{d^U}{\lambda}; h^U \right), \left(\frac{a^L}{\lambda}, \frac{b^L}{\lambda}, \frac{c^L}{\lambda}, \frac{d^L}{\lambda}; h^L \right) \right], & \text{if } \lambda > 0; \\ \left[\left(\frac{d^U}{\lambda}, \frac{c^U}{\lambda}, \frac{b^U}{\lambda}, \frac{a^U}{\lambda}; h^U \right), \left(\frac{d^L}{\lambda}, \frac{c^L}{\lambda}, \frac{b^L}{\lambda}, \frac{a^L}{\lambda}; h^L \right) \right], & \text{if } \lambda < 0. \end{cases} \quad (3)$$

It is obvious that IT2 FSs is the simplest form of T2 FSs. In this paper, we just discuss the TODIM method under IT2 FSs environment.

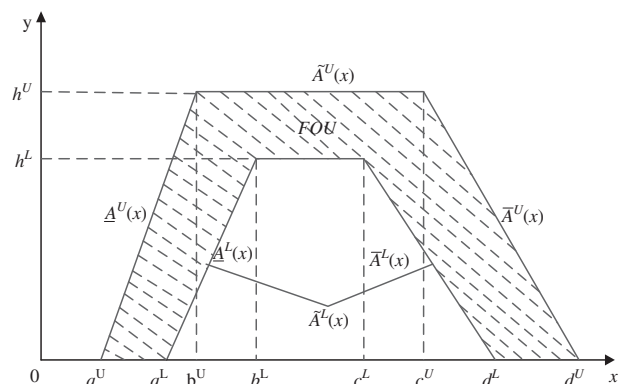


Figure 1 The sample of IT2 FSs.

2.2. Ranking method for IT2 FSs

In this section, we introduce the IT2 FSs ranking methods with possibility mean and variation coefficient concepts, which not only correctly compare the IT2 FSs especially for the symmetric IT2 FSs, but also reasonably rank the order of their images.

Definition 5 (Sang *et al.*, 2014) For an arbitrary IT2 FSs $A = [(a^U, b^U, c^U, d^U; h^U), (a^L, b^L, c^L, d^L; h^L)]$, the possibility mean of which is defined as:

$$M(\tilde{A}) = \frac{M(\tilde{A}^U) + M(\tilde{A}^L)}{2}, \quad (4)$$

where the possibility mean values of the UMF and LMF are written as:

$$M(\tilde{A}^U) = \frac{1}{2} \int_0^{h^U} (\underline{A}^U(\alpha) + \overline{A}^U(\alpha) + b^U + c^U) f(\alpha) d\alpha, \quad (5)$$

$$M(\tilde{A}^L) = \frac{1}{2} \int_0^{h^L} (\underline{A}^L(\alpha) + \overline{A}^L(\alpha) + b^L + c^L) f(\alpha) d\alpha, \quad (6)$$

$f(r)$ is an increasing function satisfying $f(0)=0, f(1)=1$ and $\int_0^{h^U} f(\alpha) d\alpha = 1/2$.

Definition 6 (Sang *et al.*, 2014) For any IT2 FSs $A = [(a^U, b^U, c^U, d^U; h^U), (a^L, b^L, c^L, d^L; h^L)]$, the variation coefficient of which is defined as:

$$VC(\tilde{A}) = \begin{cases} \frac{D(\tilde{A})}{M(\tilde{A})}, & \text{if } M(\tilde{A}) \neq 0, \\ \frac{D(\tilde{A})}{\epsilon}, & \text{if } M(\tilde{A}) = 0. \end{cases} \quad (7)$$

where ϵ is an extremely small value to present the approximate $M(\tilde{A})$, $D(\tilde{A})$ is the variation value. The expression of $D(\tilde{A})$ is defined as:

$$D(\tilde{A}) = \sqrt{D(\tilde{A}^U)D(\tilde{A}^L)}, \quad (8)$$

and

$$D(\tilde{A}^U) = \frac{1}{4} \int_0^{h^U} (\overline{A}^U(\alpha) + c^U - \underline{A}^U(\alpha) - b^U)^2 f(\alpha) d\alpha, \quad (9)$$

$$D(\tilde{A}^L) = \frac{1}{4} \int_0^{h^L} (\overline{A}^L(\alpha) + c^L - \underline{A}^L(\alpha) - b^L)^2 f(\alpha) d\alpha, \quad (10)$$

where $f(\alpha)$ is an increasing function satisfying $f(0)=0, f(1)=1$ and $\int_0^{h^U} f(\alpha) d\alpha = 1/2$.

The ordering rules for IT2 FSs \tilde{A} and \tilde{B} are carried out as follows.

Definition 7 (Sang *et al.*, 2014) Let \tilde{A} and \tilde{B} be two IT2 FSs, the comparison criteria is defined as follows:

- (1) If $M(\tilde{A}) < M(\tilde{B})$, then $\tilde{A} \prec \tilde{B}$
- (2) If $M(\tilde{A}) > M(\tilde{B})$, then $\tilde{A} \succ \tilde{B}$
- (3) If $M(\tilde{A}) = M(\tilde{B})$, then
 - (a) if $VC(\tilde{A}) < VC(\tilde{B})$, then $\tilde{A} \prec \tilde{B}$
 - (b) if $VC(\tilde{A}) > VC(\tilde{B})$, then $\tilde{A} \succ \tilde{B}$
 - (c) else $\tilde{A} \sim \tilde{B}$

It is denoted that $>$ means 'larger than' in the sense of ranking, $<$ means 'smaller than' in the sense of ranking, \sim means 'the same rank'.

2.3. The development of TODIM method

The TODIM method, proposed by Gomes and Lima (1992), is one of the first MCDM methods to calculate the dominance of one alternative over another based on non-linear prospect function, the shape of which is the same with the gains (losses) function of prospect theory (Tversky and Kahneman, 1992).

Suppose the MCDM problem has m alternatives $A_1 - m$ and n decision criteria $C_1 - n$, x_{ji} is the crisp rating of alternative A_j for criteria C_i , w_i is the crisp weight for criteria C_i satisfying $0 \leq w_i \leq 1$ and $\sum_{i=1}^n w_i = 1$. Then, the application of TODIM method in MCDM problems can be summarized in the following procedure:

- (1) Define and normalize the decision matrix $\bar{X} = (x_{ji})_{m \times n}$.
- (2) Compute the trade-off weighting factor w_{ir} between the generic criteria C_i and reference criterion C_r using Equation (11).

$$w_{ir} = \frac{w_i}{w_r} \quad (11)$$

- (3) Calculate the final dominance degree of alternative A_j over alternative A_i using Equation (12).

$$\delta(A_j, A_i) = \sum_c \phi_c(A_j, A_i), \forall (j, i), \quad (12)$$

where

$$\phi_c(A_j, A_i) = \begin{cases} \sqrt{\frac{w_{rc}}{\sum_c w_{rc}} (x_{jc} - x_{ic})}, & \text{if } x_{jc} \geq x_{ic}, \\ -\frac{1}{\theta} \sqrt{\frac{\sum_c w_{rc}}{w_{rc}}} (x_{ic} - x_{jc}), & \text{otherwise,} \end{cases} \quad (13)$$

where $\phi_c(A_j, A_i)$ represents the gains (losses) of criteria C_i to the dominance function $\phi_c(A_j, A_i)$ comparing the alternative A_j with alternative A_i , and attenuation factor θ controls the impact of loss.

- (4) Compute the normalized global performance ε_j for alternative A_j .

$$\varepsilon_j = \frac{\sum_j \delta(A_j, A_i) - \min_j \sum_i \delta(A_j, A_i)}{\max_j \sum_i \delta(A_j, A_i) - \min_j \sum_i \delta(A_j, A_i)} \quad (14)$$

- (5) Sort the alternatives according to the performance ε_j .

Later, Krohling and de Souza (2012) extended the TODIM method to trapezoidal fuzzy environment, and Equation (13) was changed into the following form as Equation (15).

$$\phi_c(A_j, A_i) = \begin{cases} \sqrt{w_{ic}}(S_{jc} - S_{ic}), & \text{if } m(S_{jc}) \geq m(S_{ic}), \\ -\frac{1}{\theta} \sqrt{w_{id}}(S_{ic} - S_{jc}), & \text{otherwise,} \end{cases} \quad (15)$$

where $m(S_{jc})$ and $m(S_{ic})$ stands for the defuzzified values of the trapezoidal fuzzy number S_{jc} and S_{ic} , respectively.

In addition, based on the Hellinger distance, Lourenzutti and Krohling (2014) revised Equation (15) into Equations (16) and (17), respectively.

If C_i is the benefit criteria, then

$$\phi_c(A_j, A_i) = \begin{cases} \sqrt{w_c} HD(f_{jc}, f_{ic}), & \text{if } D_{f_{jc} > f_{ic}} > D_{f_{ic} > f_{jc}}, \\ -\frac{1}{\theta} \sqrt{w_c} HD(f_{jc}, f_{ic}), & \text{if } D_{f_{jc} > f_{ic}} < D_{f_{ic} > f_{jc}}, \\ 0, & \text{otherwise.} \end{cases} \quad (16)$$

If C_i is the cost criteria, then

$$\phi_c(A_j, A_i) = \begin{cases} -\frac{1}{\theta} \sqrt{w_c} HD(f_{jc}, f_{ic}), & \text{if } D_{f_{jc} > f_{ic}} > D_{f_{ic} > f_{jc}}, \\ \sqrt{w_c} HD(f_{jc}, f_{ic}), & \text{if } D_{f_{jc} > f_{ic}} < D_{f_{ic} > f_{jc}}, \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

It is denoted that the Hellinger distance between f_{jc} and f_{ic} ($HD(f_{jc}, f_{ic})$) is given by

$$HD(f_{jc}, f_{ic}) = \sqrt{\frac{1}{2} \int_R (\sqrt{f_{jc}(x)} - \sqrt{f_{ic}(x)})^2 dx}.$$

3. Fuzzy decision making with interval type-2 TODIM method

In this section, we first introduce the distance computation method to count the distance between the IT2 FSs, and then propose the TODIM method under IT2 FSs environment.

3.1. The distance computation method for IT2 FSs

Due to the fact that the distance measure can be applied to many areas such as cluster analysis (Yang and Shih, 2001), approximate reasoning (Chen and Hsiao, 2000) and decision making (Wu and Mendel, 2008), but there are few research on IT2 FSs. Therefore, it is necessary to develop the distance measure under IT2 FSs environment.

Definition 8 Suppose the two IT2 FSs $\tilde{A} = [(a^U, b^U, c^U, d^U; h^U), (a^L, b^L, c^L, d^L; h^L)]$ and $\tilde{B} = [(e^U, f^U, g^U, k^U; h^U), (e^L, f^L, g^L, k^L; h^L)]$, the distance between them is defined as:

$$d(\tilde{A}, \tilde{B}) = \frac{\left(\int_0^{h^U} (|\bar{B}^U(\alpha) - \bar{A}^U(\alpha)|^r + |\underline{B}^U(\alpha) - \underline{A}^U(\alpha)|^r) d\alpha \right)^{1/r}}{2 |d^U + k^U - a^U - e^U|} + \frac{\left(\int_0^{h^L} (|\bar{B}^L(\alpha) - \bar{A}^L(\alpha)|^r + |\underline{B}^L(\alpha) - \underline{A}^L(\alpha)|^r) d\alpha \right)^{1/r}}{2 |d^L + k^L - a^L - e^L|}, \quad (18)$$

where parameter $r \in [1, +\infty)$.

In Equation (18), the proposed distance computing method can represent the distance between any IT2 FSs, especially for the symmetric IT2 FSs having different span lengths in the bottom.

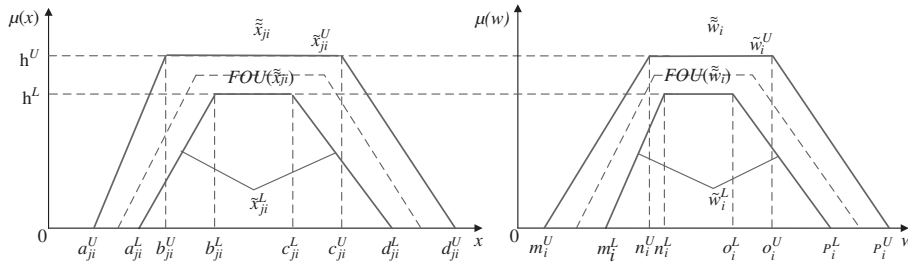
Remark 1 Regarding Equation (18), with different values of parameter r , the distance of IT2 FSs can be transformed into the following forms.

- (1) When $r = 1$, we obtain the Hamming distance for IT2 FSs, and Equation (18) can be rewritten as:

$$d(\tilde{A}, \tilde{B}) = \frac{\int_0^{h^U} (|\bar{B}^U(\alpha) - \bar{A}^U(\alpha)| + |\underline{B}^U(\alpha) - \underline{A}^U(\alpha)|) d\alpha}{2 |d^U + k^U - a^U - e^U|} + \frac{\int_0^{h^L} (|\bar{B}^L(\alpha) - \bar{A}^L(\alpha)| + |\underline{B}^L(\alpha) - \underline{A}^L(\alpha)|) d\alpha}{2 |d^L + k^L - a^L - e^L|}. \quad (19)$$

- (2) When $r = 2$, we obtain the Euclidean distance for IT2 FSs, and Equation (18) can be revised as:

$$d(\tilde{A}, \tilde{B}) = \frac{\left(\int_0^{h^U} (|\bar{B}^U(\alpha) - \bar{A}^U(\alpha)|^2 + |\underline{B}^U(\alpha) - \underline{A}^U(\alpha)|^2) d\alpha \right)^{\frac{1}{2}}}{2 |d^U + k^U - a^U - e^U|} + \frac{\left(\int_0^{h^L} (|\bar{B}^L(\alpha) - \bar{A}^L(\alpha)|^2 + |\underline{B}^L(\alpha) - \underline{A}^L(\alpha)|^2) d\alpha \right)^{\frac{1}{2}}}{2 |d^L + k^L - a^L - e^L|}. \quad (20)$$


 Figure 2 The IT2 FSs of \tilde{x}_{ji} and \tilde{w}_i .

(3) When $r \rightarrow +\infty$, Equation (18) can be transformed as:

$$\begin{aligned} d(\tilde{A}, \tilde{B}) &= \lim_{r \rightarrow +\infty} \frac{\left(\int_0^{h^U} (|\bar{B}^U(\alpha) - \bar{A}^U(\alpha)|^r + |\underline{B}^U(\alpha) - \underline{A}^U(\alpha)|^r) d\alpha \right)^{\frac{1}{r}}}{2 |d^U + k^U - a^U - e^U|} \\ &+ \lim_{r \rightarrow +\infty} \frac{\left(\int_0^{h^L} (|\bar{B}^L(\alpha) - \bar{A}^L(\alpha)|^r + |\underline{B}^L(\alpha) - \underline{A}^L(\alpha)|^r) d\alpha \right)^{\frac{1}{r}}}{2 |d^L + k^L - a^L - e^L|} \\ &= \frac{\max\{|k^U - d^U|, |g^U - c^U|, |f^U - b^U|, |e^U - a^U|\}}{2 |d^U + k^U - a^U - e^U|} \\ &+ \frac{\max\{|k^L - d^L|, |g^L - c^L|, |f^L - b^L|, |e^L - a^L|\}}{2 |d^L + k^L - a^L - e^L|}. \end{aligned} \quad (21)$$

Next, we will give some properties of the proposed distance computation method for IT2 FSs.

Property 1 Suppose \tilde{A} , \tilde{B} and \tilde{C} be any IT2 FSs, then

1. (Non-negative): $d(\tilde{A}, \tilde{B}) \geq 0$
2. (Symmetry): $d(\tilde{A}, \tilde{B}) = d(\tilde{B}, \tilde{A})$
3. (Triangle inequality): $d(\tilde{A}, \tilde{B}) + d(\tilde{B}, \tilde{C}) \geq d(\tilde{A}, \tilde{C})$
4. (Reflexivity): $d(\tilde{A}, \tilde{A}) = 0$

It can easily be seen that the proposed distance computation method also satisfies the axioms of general distance measurement (Liu, 1992).

3.2. The IT2 FSs-based TODIM method

Suppose \tilde{x}_{ji} and \tilde{w}_i are the normalized IT2 FSs, where $\tilde{x}_{ji} \in [\tilde{x}_{ji}^U(\alpha_j), \tilde{x}_{ji}^L(\alpha_j)]$, $\tilde{w}_i \in [\tilde{w}_i^U(\alpha_j), \tilde{w}_i^L(\alpha_j)]$ and the membership value of $\tilde{x}_{ji}^L(\alpha)(\tilde{w}_i^L(\alpha))$ and $\tilde{x}_{ji}^U(\alpha)(\tilde{w}_i^U(\alpha))$ is, respectively, denoted as $h_{\tilde{x}_{ji}(\alpha)}^L$ ($h_{\tilde{w}_i(\alpha)}^L$) and $h_{\tilde{x}_{ji}(\alpha)}^U$ ($h_{\tilde{w}_i(\alpha)}^U$), which are shown in Figure 2.

Thus, the fuzzy MCDM problem with IT2 FSs can be expressed as follows:

$$\begin{array}{c} C_1 \quad C_2 \quad \dots \quad C_n \\ A_1 \quad \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \end{bmatrix} \\ \tilde{X} = A_2 \quad \begin{bmatrix} \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \end{bmatrix} \\ \vdots \\ A_m \quad \begin{bmatrix} \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{array}$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n]^T,$$

where $A_1 - A_m$ are alternatives, $C_1 - C_n$ are evaluation criteria, \tilde{x}_{ji} is the rating of alternative A_j for criteria $C_i(\tilde{x}_{ji})(i = 1, 2, \dots, n; j = 1, 2, \dots, m)$, \tilde{w}_i is the fuzzy weight for criteria $C_i(i = 1, 2, \dots, n)$.

Let $\tilde{x}_{ji} = [(a_{ji}^U, b_{ji}^U, c_{ji}^U, d_{ji}^U; h_{ji}^U), (a_{ji}^L, b_{ji}^L, c_{ji}^L, d_{ji}^L; h_{ji}^L)]$ be IT2 FSs, then the normalized process can be conducted by

$$\tilde{x}_{ji} = \left[\left(\frac{a_{ji}^U}{d_{ji}^{*U}}, \frac{b_{ji}^U}{d_{ji}^{*U}}, \frac{c_{ji}^U}{d_{ji}^{*U}}, \frac{d_{ji}^U}{d_{ji}^{*U}}; h_{ji}^U \right), \left(\frac{a_{ji}^L}{d_{ji}^{*L}}, \frac{b_{ji}^L}{d_{ji}^{*L}}, \frac{c_{ji}^L}{d_{ji}^{*L}}, \frac{d_{ji}^L}{d_{ji}^{*L}}; h_{ji}^L \right) \right], j = 1, 2, \dots, m, i \in \Omega_b, \quad (22)$$

$$\tilde{x}_{ji} = \left[\left(\frac{a_{ji}^{-L}}{d_{ji}^U}, \frac{a_{ji}^{-L}}{c_{ji}^U}, \frac{a_{ji}^{-L}}{b_{ji}^U}, \frac{a_{ji}^{-L}}{d_{ji}^U}; h_{ji}^U \right), \left(\frac{a_{ji}^{-L}}{d_{ji}^L}, \frac{a_{ji}^{-L}}{c_{ji}^L}, \frac{a_{ji}^{-L}}{b_{ji}^L}, \frac{a_{ji}^{-L}}{d_{ji}^L}; h_{ji}^L \right) \right], j = 1, 2, \dots, m, i \in \Omega_c, \quad (23)$$

where Ω_b is the set of benefit criteria, Ω_c is the set of cost criteria and

$$d_{ji}^{*U} = \max_j d_{ji}^U, i \in \Omega_b, \quad a_{ji}^{-L} = \min_j a_{ji}^L, i \in \Omega_c.$$

Considering the computation complexity of IT2 FSs, we use the ranking computation method combining the possibility mean with variation coefficient for IT2 FSs and the distance

computation method for IT2 FSs to count the prospect values. Then, the gains and losses computing expression of Equation (13) can be transformed into the following form as Equations (24) and (25), respectively.

(1) For benefit criteria

$$\phi_c(A_j, A_i) = \begin{cases} \sqrt{w_c d(\tilde{x}_{jc}, \tilde{x}_{ic})}, & \text{if } \text{Rank}(\tilde{x}_{jc}) > \text{Rank}(\tilde{x}_{ic}), \\ -\frac{1}{\theta} \sqrt{w_c d(\tilde{x}_{ic}, \tilde{x}_{jc})}, & \text{if } \text{Rank}(\tilde{x}_{jc}) < \text{Rank}(\tilde{x}_{ic}), \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

(2) For cost criteria

$$\phi_c(A_j, A_i) = \begin{cases} -\frac{1}{\theta} \sqrt{w_c d(\tilde{x}_{ic}, \tilde{x}_{jc})}, & \text{if } \text{Rank}(\tilde{x}_{jc}) > \text{Rank}(\tilde{x}_{ic}), \\ \sqrt{w_c d(\tilde{x}_{jc}, \tilde{x}_{ic})}, & \text{if } \text{Rank}(\tilde{x}_{jc}) < \text{Rank}(\tilde{x}_{ic}), \\ 0, & \text{otherwise.} \end{cases} \quad (25)$$

where θ is attenuation factor; $d(\tilde{x}_{jc}, \tilde{x}_{ic})$ is the distance of IT2 FSs \tilde{x}_{jc} and \tilde{x}_{ic} computed by Equations (4) and (7); $\text{Rank}(\tilde{x}_{ic})$ is the ranking values of IT2 FSs \tilde{x}_{ic} counted by Equation (18).

Substituting the results of Equations (24) and (25) into Equation (12), it is obvious that the form of the dominance values $\delta(A_j, A_i)$ is IT2 FSs. To select the best alternative, the dominance values need to be defuzzified. The ranking values proposed by Sang *et al* (2014) perhaps is a suitable defuzzification method to represent the fuzzy information and therefore is utilized in this paper.

Then, the global performance for the alternative in Equation (14) can be correspondingly revised as Equation (26).

$$\varepsilon_j = \frac{\text{Rank}\left(\sum_i \delta(A_j, A_i)\right) - \min_j \left(\text{Rank}\left(\sum_i \delta(A_j, A_i)\right)\right)}{\max_j \left(\text{Rank}\left(\sum_i \delta(A_j, A_i)\right)\right) - \min_j \left(\text{Rank}\left(\sum_i \delta(A_j, A_i)\right)\right)} \quad (26)$$

By combining with the presented IT2 FSs ranking method and the proposed distance computation method together, the procedure of IT2 FSs-based TODIM method can be summarized as follows:

- Step 1:** Construct and normalize the fuzzy decision matrix $\tilde{X} = (\tilde{x}_{ji})_{m \times n}$ by Equations (22) and (23), respectively.
- Step 2:** Construct and count the fuzzy average weighting matrix \tilde{W} for each criteria provided by the decision maker.

- Step 3:** Calculate the gains and loss of alternative A_j over the alternative A_i regarding criteria C_i with Equations (24) or (25).
- Step 4:** Compute the dominance values of alternative A_j over alternative A_i using Equation (12).
- Step 5:** Calculate the ranking values of the dominance degree for alternative A_j over alternative A_i with Equations (4) and (7).
- Step 6:** Compute the normalized global performance ε_j for alternative A_j using Equation (26).
- Step 7:** Rank and sort the alternatives according to the performance ε_j .

Remark 2 When computing the gains and losses with Equations (24) and (25), the distance computation expression can be specified with corresponding parameter values, which provides multiple choices for IT2 FSs-based TODIM models.

The proposed IT2 FSs-based TODIM method provides a new way to solve MCDM problems under uncertainty environment rather than the current fuzzy TODIM method and crisp TODIM methods, which uses the form of IT2 FSs to represent the evaluations and the weights of attributes.

4. The application of green supplier selection in an automobile manufacturer

Suppose an automobile manufacturer want to choose the proper green supplier from three alternatives A_1 – A_3 . There are three decision makers (D_1 – D_3) to evaluate the five alternatives, and the assessment criteria to be conducted are: price (C_1), quality (C_2), delivery on time (C_3), service (C_4), technology (C_5), corporate social responsibility (C_6) and environment (C_7).

The evaluation of the criteria are defined in Table 1. Tables 2 and 3 show the assessments and weights provided by the three decision makers, where aggregated fuzzy numbers are obtained by averaging the fuzzy opinions of the three decision makers. That is $\tilde{x}_{ji} = (1)/(3) \sum_{k=1}^3 \tilde{x}_{ji}^k$ and $\tilde{w}_i = (1)/(3) \sum_{k=1}^3 \tilde{w}_i^k$, where \tilde{x}_{ji}^k and \tilde{w}_i^k are the average rating and weight given by the k th decision maker, respectively.

4.1. Computing process

Suppose the parameter $r = 1$ in computing the distance between IT2 FSs and the attenuation factor $\theta = 1$ in calculating the dominance degree, the procedure of evaluating and selecting the best supplier for the automobile manufacturer can be listed as below:

- Step 1:** Construct the decision matrix \tilde{X} , and normalize the fuzzy average decision matrix, which is shown in Table 2.

Note that we have utilized Equations (22) and (23) to normalize the benefit criteria and the cost criteria, respectively.

Step 2: Construct the weighting matrix \tilde{W}_p , and normalize average weighting matrix as $\tilde{\bar{W}} = (\tilde{w}_i)_{1 \times n}$, which is shown in Table 3.

Here, as the weights \tilde{w}_i for criteria \tilde{C}_i are normalized IT2 FSs linguistic variables, we can directly use the average IT2 FSs weights to compute the gains (losses).

Step 3: Compute the ranking values of evaluation in fuzzy average decision matrix using Equations (4) and (7), which are shown in Table 4.

Step 4: Count the distance between evaluations using Equation (18), which are listed in Table 5.

Step 5: Calculate the gains (losses) between alternative A_1 over alternative A_2 .

Table 1 Linguistic variables for the evaluation of criteria

Linguistic variable	IT2 FSs
Very Low (VL)	[(0, 0, 0, 0.1;1), (0, 0, 0, 0.05;0.9)]
Low (L)	[(0, 0.1, 0.1, 0.3;1), (0.05, 0.1, 0.1, 0.2;0.9)]
Medium Low (ML)	[(0.1, 0.3, 0.3, 0.5;1), (0.2, 0.3, 0.3, 0.4;0.9)]
Medium (M)	[(0.3, 0.5, 0.5, 0.7;1), (0.4, 0.5, 0.5, 0.6;0.9)]
Medium High (MH)	[(0.5, 0.7, 0.7, 0.9;1), (0.6, 0.7, 0.7, 0.8;0.9)]
High (H)	[(0.7, 0.9, 0.9, 1;1), (0.8, 0.9, 0.9, 0.95;0.9)]
Very High (VH)	[(0.9, 1, 1, 1;1), (0.95, 1, 1, 1;0.9)]

Table 2 The evaluation of the three candidates by all decision makers

Criteria	Alternatives	Decision makers			Average IT2 FSs
		D_1	D_2	D_3	
Price(C_1)	A_1	MH	H	MH	[(0.61, 0.74, 0.74, 1;1), (0.67, 0.74, 0.74, 0.85;0.9)]
	A_2	H	MH	H	[(0.59, 0.68, 0.68, 0.89;1), (0.63, 0.68, 0.68, 0.77;0.9)]
	A_3	VH	H	MH	[(0.59, 0.65, 0.65, 0.81;1), (0.62, 0.65, 0.65, 0.72;0.9)]
Quality(C_2)	A_1	H	VH	H	[(0.7, 0.75, 0.75, 1;1), (0.72, 0.75, 0.75, 0.82;0.9)]
	A_2	MH	H	VH	[(0.72, 0.8, 0.8, 1;1), (0.76, 0.8, 0.8, 0.9;0.9)]
	A_3	VH	VH	H	[(0.7, 0.72, 0.72, 1;1), (0.71, 0.72, 0.72, 0.78;0.9)]
Delivery on time(C_3)	A_1	VH	H	H	[(0.77, 0.93, 0.93, 1;1), (0.85, 0.93, 0.93, 0.97;0.9)]
	A_2	H	VH	VH	[(0.83, 0.97, 0.97, 1;1), (0.9, 0.97, 0.97, 0.98;0.9)]
	A_3	M	MH	MH	[(0.43, 0.63, 0.63, 0.83;1), (0.53, 0.63, 0.63, 0.73;0.9)]
Service(C_4)	A_1	VH	H	H	[(0.77, 0.93, 0.93, 1;1), (0.85, 0.93, 0.93, 0.97;0.9)]
	A_2	H	VH	H	[(0.83, 0.97, 0.97, 1;1), (0.9, 0.97, 0.97, 0.98;0.9)]
	A_3	H	VH	VH	[(0.77, 0.93, 0.93, 1;1), (0.85, 0.93, 0.93, 0.97;0.9)]
Capability(C_5)	A_1	VH	H	H	[(0.77, 0.93, 0.93, 1;1), (0.85, 0.93, 0.93, 0.97;0.9)]
	A_2	VH	VH	VH	[(0.9, 1, 1, 1;1), (0.95, 1, 1, 1;0.9)]
	A_3	H	VH	VH	[(0.83, 0.97, 0.97, 1;1), (0.9, 0.97, 0.97, 0.98;0.9)]
Reputation(C_6)	A_1	H	H	H	[(0.7, 0.9, 0.9, 1;1), (0.8, 0.9, 0.9, 0.95;0.9)]
	A_2	H	H	H	[(0.7, 0.9, 0.9, 1;1), (0.8, 0.9, 0.9, 0.95;0.9)]
	A_3	H	VH	VH	[(0.83, 0.97, 0.97, 1;1), (0.9, 0.97, 0.97, 0.98;0.9)]
Technology(C_7)	A_1	VH	H	H	[(0.77, 0.93, 0.93, 1;1), (0.85, 0.93, 0.93, 0.97;0.9)]
	A_2	H	MH	H	[(0.63, 0.83, 0.83, 0.97;1), (0.73, 0.83, 0.83, 0.9;0.9)]
	A_3	H	MH	MH	[(0.57, 0.77, 0.77, 0.93;1), (0.67, 0.77, 0.77, 0.85;0.9)]

According to Equations (24) and (25), the gains (losses) of criteria C_i to function $\delta(A_i, A_j)$ named $\phi_c(A_i, A_j)$ are shown in Tables 6–11.

Step 6: Compute the dominance values between the alternatives.

Substitute the values of Tables 6–11 into Equation (12), the dominance values of which are shown in Table 12.

Step 7: Compute the global performance for alternative A_j .

(a) Calculate the ranking values of the dominance degree between alternatives. Using Equations (4)–(6), the ranking values of $\Sigma_j \delta(A_i, A_j)$ can be obtained as:

$$\begin{aligned} & \text{Rank}(\Sigma_j \delta(A_1, A_j)) \\ &= \text{Rank}(\delta(A_1, A_1) + \delta(A_1, A_2) + \delta(A_1, A_3)) = -1.489 \end{aligned}$$

$$\begin{aligned} & \text{Rank}(\Sigma_j \delta(A_2, A_j)) \\ &= \text{Rank}(\delta(A_2, A_1) + \delta(A_2, A_2) + \delta(A_2, A_3)) = 4.642 \end{aligned}$$

$$\begin{aligned} & \text{Rank}(\Sigma_j \delta(A_3, A_j)) \\ &= \text{Rank}(\delta(A_3, A_1) + \delta(A_3, A_2) + \delta(A_3, A_3)) = -2.53 \end{aligned}$$

(b) Count the global performance ε_j for each alternative.

Take the ranking values of the dominance into Equation (26), and the global performance for the alternatives can be obtained as: $A_1 = 0.159$, $A_2 = 1$ and $A_3 = 0$. The order of which is shown

Table 3 The relative weights of the five criteria provided by decision makers

Criteria	D_1	D_2	D_3	Average IT2 FSs
price(C_1)	VH	H	VH	[(0.83, 0.97, 0.97, 1;1), (0.9, 0.97, 0.97, 0.98;0.9)]
quality(C_2)	H	VH	VH	[(0.83, 0.97, 0.97, 1;1), (0.9, 0.97, 0.97, 0.98;0.9)]
delivery on time(C_3)	H	H	H	[(0.7, 0.9, 0.9, 1;1), (0.8, 0.9, 0.9, 0.95;0.9)]
service(C_4)	VH	VH	MH	[(0.77, 0.9, 0.9, 0.97;1), (0.83, 0.9, 0.9, 0.93;0.9)]
capability(C_5)	VH	VH	VH	[(0.9, 1, 1, 1;1), (0.95, 1, 1, 1;0.9)]
reputation(C_6)	H	MH	MH	[(0.57, 0.77, 0.77, 0.93;1), (0.67, 0.77, 0.77, 0.85;0.9)]
technology(C_7)	MH	MH	MH	[(0.5, 0.7, 0.7, 0.9;1), (0.6, 0.7, 0.7, 0.8;0.9)]

Table 4 The ranking values of evaluation in fuzzy average decision matrix

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	0.6765	0.6884	0.8366	0.8366	0.8366	0.8087	0.8366
A_2	0.6218	0.7553	0.8712	0.8712	0.8992	0.8087	0.7476
A_3	0.6201	0.6641	0.5702	0.8366	0.8712	0.8712	0.6945

Table 5 The distance between evaluations in fuzzy average decision matrix

Distance	$d(x_{11}, x_{21})$	$d(x_{12}, x_{22})$	$d(x_{13}, x_{23})$	$d(x_{14}, x_{24})$	$d(x_{15}, x_{25})$	$d(x_{16}, x_{26})$	$d(x_{17}, x_{27})$
Value	0.2593	0.2580	0.2450	0.2450	0.5619	0	0.4649
Distance	$d(x_{11}, x_{31})$	$d(x_{12}, x_{32})$	$d(x_{13}, x_{33})$	$d(x_{14}, x_{34})$	$d(x_{15}, x_{35})$	$d(x_{16}, x_{36})$	$d(x_{17}, x_{37})$
Value	0.4491	0.1706	1.256	0	0.2450	0.4078	0.7150
Distance	$d(x_{21}, x_{31})$	$d(x_{22}, x_{32})$	$d(x_{23}, x_{33})$	$d(x_{24}, x_{34})$	$d(x_{25}, x_{35})$	$d(x_{26}, x_{36})$	$d(x_{27}, x_{37})$
Value	0.1846	0.4312	1.5929	0.2450	0.3454	0.4078	0.2264

Table 6 The gains (losses) of criteria C_i to alternative A_1 and A_2

Gains (losses)	IT2 FSs
$\phi_1(A_1, A_2)$	[(-0.46, -0.50, -0.50, -0.51;1), (-0.48, -0.50, -0.50, -0.50;0.9)]
$\phi_2(A_1, A_2)$	[(0.46, 0.50, 0.50, 0.51;1), (0.48, 0.50, 0.50, 0.50;0.9)]
$\phi_3(A_1, A_2)$	[(-0.41, -0.47, -0.47, -0.49;1), (-0.44, -0.47, -0.47, -0.48;0.9)]
$\phi_4(A_1, A_2)$	[(-0.43, -0.47, -0.47, -0.49;1), (-0.45, -0.47, -0.47, -0.48;0.9)]
$\phi_5(A_1, A_2)$	[(-0.71, -0.75, -0.75, -0.75;1), (-0.73, -0.75, -0.75, -0.75;0.9)]
$\phi_6(A_1, A_2)$	[(0, 0, 0, 0;1), (0, 0, 0, 0;0.9)]
$\phi_7(A_1, A_2)$	[(0.48, 0.57, 0.57, 0.65;1), (0.53, 0.57, 0.57, 0.61;0.9)]

Table 7 The gains (losses) of criteria C_i to alternative A_1 and A_3

Gains (losses)	IT2 FSs
$\phi_1(A_1, A_3)$	[(-0.61, -0.66, -0.66, -0.67;1), (-0.64, -0.66, -0.66, -0.66;0.9)]
$\phi_2(A_1, A_3)$	[(0.38, 0.41, 0.41, 0.41;1), (0.39, 0.41, 0.41, 0.41;0.9)]
$\phi_3(A_1, A_3)$	[(0.94, 1.06, 1.06, 1.12;1), (1.00, 1.06, 1.06, 1.09;0.9)]
$\phi_4(A_1, A_3)$	[(0, 0, 0, 0;1), (0, 0, 0, 0;0.9)]
$\phi_5(A_1, A_3)$	[(-0.47, -0.49, -0.49, -0.49;1), (-0.48, -0.49, -0.49, -0.49;0.9)]
$\phi_6(A_1, A_3)$	[(-0.48, -0.56, -0.56, -0.62;1), (-0.52, -0.56, -0.56, -0.59;0.9)]
$\phi_7(A_1, A_3)$	[(0.60, 0.71, 0.71, 0.80;1), (0.65, 0.71, 0.71, 0.76;0.9)]

as below:

$$A_2 \succ A_1 \succ A_3.$$

4.2. Sensitivity analysis

The sensitivity analysis was carried out through varying the attenuation factor θ in dominance degree formula and the parameter r in distance computing expression for IT2 FSs-based TODIM method.

- (1) The sensitivity analysis on attenuation factor θ . Suppose the attenuation factor value $\theta = 1, 2, 3$, with the similar process of computing the performance of the alternatives for $\theta = 1$, the ranking values and their orders of the three green suppliers for the automobile manufacturer are listed in Table 13. It can easily be seen that the rankings of the three green suppliers for automobile manufacturer is consistent. That is, with the change of the attenuation factor values, the orders of the three green suppliers are not sensitive to the value of θ .
- (2) The sensitivity analysis on distance computation parameter r .

Table 8 The gains (losses) of criteria C_i to alternative A_2 and A_1

Gains (losses)	IT2 FSs
$\phi_1(A_2, A_1)$	[(0.46, 0.50, 0.50, 0.51;1),(0.48, 0.50, 0.50, 0.50;0.9)]
$\phi_2(A_2, A_1)$	[(0.46, 0.50, 0.50, 0.51;1),(0.48, 0.50, 0.50, 0.50;0.9)]
$\phi_3(A_2, A_1)$	[(1.06, 1.20, 1.20, 1.26;1),(1.13, 1.20, 1.20, 1.23;0.9)]
$\phi_4(A_2, A_1)$	[(0.43, 0.47, 0.47, 0.49;1),(0.45, 0.47, 0.47, 0.48;0.9)]
$\phi_5(A_2, A_1)$	[(0.71, 0.75, 0.75, 0.75;1),(0.73, 0.75, 0.75, 0.75;0.9)]
$\phi_6(A_2, A_1)$	[(0, 0, 0, 0;1),(0, 0, 0, 0;0.9)]
$\phi_7(A_2, A_1)$	[(-0.48, -0.57, -0.57, -0.65;1), (-0.53, -0.57, -0.57, -0.61;0.9)]

Table 9 The gains (losses) of criteria C_i to alternative A_2 and A_3

Gains (losses)	IT2 FSs
$\phi_1(A_2, A_3)$	[(-0.39, -0.42, -0.42, -0.43;1), (-0.41, -0.42, -0.42, -0.43;0.9)]
$\phi_2(A_2, A_3)$	[(0.60, 0.65, 0.65, 0.66;1),(0.62, 0.65, 0.65, 0.65;0.9)]
$\phi_3(A_2, A_3)$	[(1.06, 1.20, 1.20, 1.26;1),(1.13, 1.20, 1.20, 1.23;0.9)]
$\phi_4(A_2, A_3)$	[(0.43, 0.47, 0.47, 0.49;1),(0.45, 0.47, 0.47, 0.48;0.9)]
$\phi_5(A_2, A_3)$	[(0.56, 0.59, 0.59, 0.59;1),(0.57, 0.59, 0.59, 0.59;0.9)]
$\phi_6(A_2, A_3)$	[(-0.48, -0.56, -0.56, -0.62;1), (-0.52, -0.56, -0.56, -0.59;0.9)]
$\phi_7(A_2, A_3)$	[(0.34, 0.40, 0.40, 0.45;1),(0.37, 0.40, 0.40, 0.43;0.9)]

Table 10 The gains (losses) of criteria C_i to alternative A_3 and A_1

Gains (losses)	IT2 FSs
$\phi_1(A_3, A_1)$	[(0.61, 0.66, 0.66, 0.67;1),(0.64, 0.66, 0.66, 0.66;0.9)]
$\phi_2(A_3, A_1)$	[(-0.38, -0.41, -0.41, -0.41;1), (-0.39, -0.41, -0.41, -0.41;0.9)]
$\phi_3(A_3, A_1)$	[(-0.94, -1.06, -1.06, -1.12;1), (-1.00, -1.06, -1.06, -1.09;0.9)]
$\phi_4(A_3, A_1)$	[(0, 0, 0, 0;1),(0, 0, 0, 0;0.9)]
$\phi_5(A_3, A_1)$	[(0.47, 0.49, 0.49, 0.49;1),(0.48, 0.49, 0.49, 0.49;0.9)]
$\phi_6(A_3, A_1)$	[(0.48, 0.56, 0.56, 0.62;1),(0.52, 0.56, 0.56, 0.59;0.9)]
$\phi_7(A_3, A_1)$	[(-0.60, -0.71, -0.71, -0.80;1), (-0.65, -0.71, -0.71, -0.76;0.9)]

Assume the attenuation factor $\theta = 1$ and distance computation parameter $r = 1, 2, \infty$, with the similar process of computing the performance of the alternatives for $\theta = 1$ and $r = 1$, the rankings of the three green suppliers for the automobile manufacturer are listed in Table 14.

It is concluded that the rankings of the three green suppliers for automobile manufacturer is consistent. That is to say, with varying the value of parameter, the orders of the three green suppliers are not sensitive to the value r .

4.3. Discussions

Combining the theoretical analysis and the application in green supplier selection for automobile manufacturer together, the

Table 11 The gains (losses) of criteria C_i to alternative A_3 and A_2

Gains (losses)	IT2 FSs
$\phi_1(A_3, A_2)$	[(0.39, 0.42, 0.42, 0.43;1),(0.41, 0.42, 0.42, 0.43;0.9)]
$\phi_2(A_3, A_2)$	[(-0.60, -0.65, -0.65, -0.66;1), (-0.62, -0.65, -0.65, -0.65;0.9)]
$\phi_3(A_3, A_2)$	[(-1.06, -1.20, -1.20, -1.26;1), (-1.13, -1.20, -1.20, -1.23;0.9)]
$\phi_4(A_3, A_2)$	[(-0.43, -0.47, -0.47, -0.49;1), (-0.45, -0.47, -0.47, -0.48;0.9)]
$\phi_5(A_3, A_2)$	[(-0.56, -0.59, -0.59, -0.59;1), (-0.57, -0.59, -0.59, -0.59;0.9)]
$\phi_6(A_3, A_2)$	[(0.48, 0.56, 0.56, 0.62;1),(0.52, 0.56, 0.56, 0.59;0.9)]
$\phi_7(A_3, A_2)$	[(-0.34, -0.40, -0.40, -0.45;1), (-0.37, -0.40, -0.40, -0.43;0.9)]

Table 12 The dominance values $\delta(A_i, A_j)$ between alternatives

Alternative A_i	Alternative A_j	IT2 FSs
A_1	A_1	[(0, 0, 0, 0;1),(0, 0, 0, 0;0.9)]
	A_2	[(-2.01, -2.12, -2.12, -2.10;1), (-2.06, -2.12, -2.12, -2.11;0.9)]
	A_3	[(-0.35, -0.46, -0.46, -0.55;1), (-0.41, -0.46, -0.46, -0.51;0.9)]
A_2	A_1	[(2.65, 2.85, 2.85, 2.87;1), (2.75, 2.85, 2.85, 2.86;0.9)]
	A_2	[(0, 0, 0, 0;1),(0, 0, 0, 0;0.9)]
	A_3	[(-2.11, -2.32, -2.32, -2.40;1), (-2.22, -2.32, -2.32, -2.36;0.9)]
A_3	A_1	[(0.35, 0.46, 0.46, 0.55;1), (0.41, 0.46, 0.46, 0.51;0.9)]
	A_2	[(2.11, 2.32, 2.32, 2.40;1), (2.22, 2.32, 2.32, 2.36;0.9)]
	A_3	[(0, 0, 0, 0;1),(0, 0, 0, 0;0.9)]

Table 13 The performance of the alternatives and their orders with different attenuation factor values

Attenuation (θ)	ε_{A_1}	ε_{A_2}	ε_{A_3}	Ranking order
1	0.159	1	0	$A_2 > A_1 > A_3$
2	0.113	1	0	$A_2 > A_1 > A_3$
3	0.089	1	0	$A_2 > A_1 > A_3$

Table 14 The performance of the alternatives and their orders with different distance parameter values

Parameter $r(\theta = 1)$	ε_{A_1}	ε_{A_2}	ε_{A_3}	Ranking order
1	0.159	1	0	$A_2 > A_1 > A_3$
2	0.142	1	0	$A_2 > A_1 > A_3$
$+\infty$	0.063	1	0	$A_2 > A_1 > A_3$

conclusions of the proposed IT2 FSs-based TODIM method can be summarized as below:

- (1) IT2 FSs was an extension of the ordinary fuzzy set, which is able to handle linguistic uncertainties (words can mean different things to different people) as well as numerical uncertainties, and has been found useful to deal with vagueness and uncertainty in green supplier evaluation and selection problems for automobile manufacturers.
- (2) The proposed IT2 FSs-based TODIM method considers the decision makers as partially rational, which provides a new way to solve green supplier evaluation and selection problems for automobile manufacturers under IT2 FSs environment.
- (3) From the performance of the three green supplier alternatives in Tables 13 and 14, it is obvious that green supplier A_2 is the best for this automobile manufacturer, and the ranking of the green supplier alternatives is $A_2 > A_1 > A_3$.

5. Conclusions and future research

With the increasing awareness and significant environmental pressures from various stakeholders, companies have realized the significance of selecting green suppliers to their supply chain activities, which involves multiple criteria with uncertainty and the decision makers' behaviour with irrational. For handling ambiguity evaluations and irrationality of the decision makers in MCDM problems, an IT2 FSs-based TODIM method was applied to select green suppliers for the automobile manufacturer.

Compared with the existing literature, the proposed IT2 FSs-based TODIM decision model contributes to the evaluation and selection of green suppliers. The key conclusions drawn from the research are listed as below:

- (1) Providing a new way for developing fuzzy TODIM method, due to the fact that it uses the form of IT2 FSs to represent the evaluations and the weights of attributes.
- (2) Proposing the distance computation method to calculate the distance between the IT2 FSs, constructing the gains and losses expressions for the benefit and cost criteria and obtaining the dominance values in form of IT2 FSs, and defuzzifying the performance function for the alternatives.
- (3) Applying the IT2 FSs-based TODIM model to green supplier selection problems for automobile manufacturer, and analysing the parameters sensitivity of attenuation factor and distance parameter with different values, respectively.

Although the results obtained from this research are satisfactory, there still exists room for improvement.

- (1) Extending the TODIM method to other types of preference information and weights, especially with different preference representation structures simultaneously.
- (2) Using more complex fuzzy TODIM models that allow for taking the third-generation prospect theory and stochastic fuzzy MCDM methods with different kinds of assessments and probability weights into consideration.

- (3) Developing different types of applications not only in decision theory but also in other fields, such as engineering and economics.

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